1050 - Exam 5 Review

1. $\frac{7!}{4!} = \frac{7\cdot 6\cdot 5\cdot \cancel{4\cdot 3\cdot 2\cdot 1}}{\cancel{4\cdot 3\cdot 2\cdot 1}} = \boxed{210}$

2. (a) $\{s_n\} = \{4n-2\}$

$s_1 = 4(1)-2 = \boxed{2}$

$s_2 = 4(2)-2 = \boxed{6}$

$s_3 = 4(3)-2 = \boxed{10}$

$s_4 = 4(4)-2 = \boxed{14}$

$s_5 = 4(5)-2 = \boxed{18}$

(b) $\{a_n\} = 2n^2+n$

$a_1 = 2(1)^2+1 = \boxed{3}$

$a_2 = 2(2)^2+2 = \boxed{10}$

$a_3 = 2(3)^2+3 = \boxed{21}$

$a_4 = 2(4)^2+4 = \boxed{36}$

$a_5 = 2(5)^2+5 = \boxed{55}$

3. $a_1 = \boxed{8}$ $\quad a_n = 5a_{n-1}+2$

$a_2 = 5a_1+2 = 5(8)+2 = \boxed{42}$

$a_3 = 5a_2+2 = 5(42)+2 = \boxed{212}$

$a_4 = 5a_3+2 = 5(212)+2 = \boxed{1062}$

$a_5 = 5a_4+2 = 5(1062)+2 = \boxed{5312}$

4. $\sum_{k=1}^{n}(3k-1) = 3(1)-1 + 3(2)-1 + 3(3)-1 + \ldots + 3n-1$

$= \boxed{2+5+8+\ldots+3n-1}$

5. $4^3+5^3+6^3+\ldots+13^3 = \boxed{\sum_{k=4}^{13} k^3}$

6. $\sum_{k=2}^{5}(3k+7) = 3(2)+7 + 3(3)+7 + 3(4)+7 + 3(5)+7$

$= 13+16+19+22 = \boxed{70}$

7. $a_1 = 5 \quad d = 3$

$a_n = a_1 + (n-1)d$

$a_n = 5 + (n-1)3$

$a_n = 5 + 3n - 3$

$\boxed{a_n = 3n+2}$

$a_{18} = 5 + (18-1)3$

$\boxed{a_{18} = 56}$

8. $n = 28$ $\quad a_{28} = 18 + (28-1)2$ $\quad S_n = \frac{n}{2}(a_1 + a_n)$

$a_1 = 18$ $\quad a_{28} = 72$ $\quad S_{28} = \frac{28}{2}(18+72)$

$d = 2$ $\quad = \boxed{1260 \text{ seats}}$

9. 7th term is -47, 13th term is -101

$a_7 = -47 = a_1 + 6d$

$a_{13} = -101 = a_1 + 12d$

$(a_1 + 6d = -47)(-1)$

$\cancel{-a_1} - 6d = 47$

$\cancel{a_1} + 12d = -101$

$6d = -54$

$\boxed{d = -9}$

$a_1 + 6(-9) = -47$

$\boxed{a_1 = 7}$

$\boxed{a_1 = 7 \quad a_n = a_{n-1} - 9}$

10. $(-5) + (-2) + 1 + 4 + \ldots + 76$

$a_1 = -5 \quad d = 3 \quad a_n = 76$

$a_n = a_1 + (n-1)d$

$76 = -5 + (n-1)3$

$76 = -5 + 3n - 3$

$76 = 3n - 8$

$3n = 84$

$n = 28$

$S_n = \frac{n}{2}(a_1 + a_n)$

$S_n = \frac{28}{2}(-5 + 76)$

$S_n = \boxed{994}$

11. 10th term of $-1, \frac{1}{2}, -\frac{1}{4}, \ldots$

$a_1 = -1, \quad r = -\frac{1}{2}$

$a_n = a_1 r^{n-1}$

$a_{10} = -1 \cdot (-\frac{1}{2})^{10-1} = -1 \cdot -\frac{1}{512} = \boxed{\frac{1}{512}}$

12. $a = 3 \quad r = 4$

$\boxed{a_n = 3 \cdot 4^{n-1}}$

$a_6 = 3 \cdot 4^{6-1} = \boxed{3072}$

13. $2, 6, 18, 54, 162, \ldots$

$a_1 = 2 \quad r = 3$ $\quad \boxed{a_n = 2 \cdot 3^{n-1}}$

14. $\sum_{k=1}^{5} (1/2)(2)^k = \sum_{k=1}^{5} (1/2)(2)(2)^{k-1} = \sum_{k=1}^{5} 2^{k-1}$

$a_1 = 1 \quad r = 2 \qquad S_5 = 1 \cdot \frac{1-2^5}{1-2} = \boxed{31}$

$S_n = a_1 \cdot \frac{1-r^n}{1-r}$

15. $\sum_{k=1}^{\infty} \frac{2}{3} \cdot 2^{k-1} \quad r = 2 \quad |r| > 1 \quad \boxed{\text{diverges}}$

(b) $4 - 2 + 1 - 1/2 + \ldots \qquad S = \frac{a_1}{1-r}$

$a_1 = 4 \quad r = -1/2 \qquad S = \frac{4}{1-(-1/2)}$

$|r| < 1 \quad \boxed{\text{converges}} \qquad \boxed{S = 8/3}$

16. (a) $\{-5n+2\}$

$a_n = -5n+2$

$a_{n-1} = -5(n-1)+2 = -5n+5+2 = -5n+7$

$a_n - a_{n-1} = (-5n+2)-(-5n+7)$

$= -5n+2+5n-7 = -5 \leftarrow$ constant

$\boxed{\text{arithmetic}}$

(b) $\{4n^2+7\}$

$a_n = 4n^2+7$

$a_{n-1} = 4(n-1)^2+7$

$= 4(n^2-2n+1)+7$

$= 4n^2-8n+4+7$

$= 4n^2-8n+11$

$a_n - a_{n-1} =$

$(4n^2+7)-(4n^2-8n+11) =$

$4n^2+7-4n^2+8n-11 =$

$8n-4 \leftarrow$ not constant

$\boxed{\text{not arithmetic}}$

$a_n/a_{n-1} = (4n^2+7)/(4n^2-8n+11)$ ↙

$\boxed{\text{not geometric}}$

(c) $\{3^{2n}\} \quad a_n = 3^{2n} = 9^n \quad a_{n-1} = 3^{2(n-1)} = 9^{n-1}$

$a_n - a_{n-1} = 9^n - 9^{n-1} \leftarrow$ not constant $\boxed{\text{not arithmetic}}$

$a_n/a_{n-1} = 9^n/9^{n-1} = 9^{n-(n-1)} = 9^1 = 9 \leftarrow$ constant

$\boxed{\text{geometric}}$